

# Intermagnitude Relationships and Asperity Statistics

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*Abstract*—Several data sources appeared recently which enable one to construct an updated version of global average intermagnitude relationships. A set of nonlinear magnitude vs  $M_w$  (or  $\log M_o$ ) curves is presented. Several regional scales are also included in the set. Utilization of  $M_w$  as a referential scale provides the optimal basis for extrapolation of return periods, strong motion amplitudes and source parameters from moderate to large earthquakes. Remarkable features of the constructed curves are: (1) practical coincidence of modified  $m_b$  ( $m_b^*$  or  $\hat{m}_b$ ) scale with  $m^{\text{SKM}}$  of the Soviet Seismological Service (except the constant shift of 0.18); (2) the lack of true saturation of all scales but  $m_b$  with the possible exception of  $m_B$  for  $M_w > 8.7$ ; (3) almost common shape of curves for all short-period magnitudes (global as well as regional); (4) lack of a systematic world-averaged difference between American and Soviet  $M_s$ .

For  $M_o$  large enough,  $m^{\text{SKM}}$  (or  $m_b^*$ ) vs  $\log M_o$  trend has the slope  $b = 0.35$ . Data are compiled to estimate short-period spectral level vs  $\log M_o$  trend, and its slope seems to be near  $\beta = 0.39$ . These two values and some additional assumptions, rather common, lead to the following conclusions regarding properties of the earthquake source and its short-period radiation: (1) peak to rms amplitude ratio (peak factor) of a short-period record grows as  $M_o^{0.11-0.13}$ , hence the Gaussian noise model of the record (predicting  $M_o^{0.03-0.04}$ ) can be definitely rejected, and some heavy-tailed peak distribution is to be assumed instead; (2) if one specifies this distribution to be the power-law (Pareto), then the estimate of the exponent  $\alpha$  for this distribution approaches 2 for short-period teleseismic records (and this  $\alpha$  value is the same as had been found previously for accelerogram peaks); (3) this may indicate the power law distribution of seismic force values of individual asperities; (4) there is a difference between our estimated  $\beta = 0.39$  and  $\beta = 1/3$  expected from the  $\omega^{-2}$  model; data are not sufficient at present to show definitely that the difference is real.

**Key words:** Magnitude, moment magnitude, saturation of magnitude, peak factor, asperity, seismic force, Pareto distribution.

## Introduction

Many practical problems of seismology require conversion between different magnitude scales. Empirical intermagnitude relationships were initially approximated as linear but detailed investigations revealed nonlinear relationships in most cases. UTSU (1982) systematically compared magnitudes based on moment magnitude  $M_w = 2/3 \log M_o - 10.7$  (HANKS and KANAMORI, 1979) as the fundamental

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one. GUSEV (1983) used nonlinear  $M - \log M_o$  relations to study spectral trends. Data acquired presently enable the improvement of these results. Routine  $M_o$  determinations by the Harvard group (DZIEWONSKI and WOODHOUSE, 1983) and the NEIC group (SIPKIN, 1986) provided additional data for the derivation of updated intermagnitude relations for small to moderate events. For large events, a very useful data set was compiled by PURCARU and BERCKHEMER (1982). The data set published by ABE (1981, 1984) and ABE and NOGUCHI (1983a,b) is also very important. The four main data sets will be denoted below as HAR, NEIC, PB and A. Conventional catalogues of the Soviet ESSN and American NEIC services are used as well, and some published intermagnitude relations are also included in compilation.

Throughout the paper we use the  $M_w$  (or  $M_o$ ) scale as the basic one. This scale is physically transparent, and predictions of fault/source parameters and of strong motion amplitudes based on it have more chances for success. Below, we shall limit our study to shallow focus earthquakes only; moreover, condition  $h < 50$  km was applied when studying  $M_s$ .

The results of our study of intermagnitude relationships are mainly of technical value. Indeed, they provide powerful tools to study radiation properties of "average" earthquakes. In the concluding part of the paper we show some implications of such an approach.

### *A Technique to Estimate Intermagnitude Relationships. Notation*

To determine a linear intermagnitude relationship, a method of orthogonal regression is normally used. No standard nonlinear generalization of this technique was at our disposal, thus we employed the following procedure (designed to imitate the orthogonal regression locally): (1) choose the scales of the plot of two magnitudes ( $M_1$  vs  $M_2$ ) so that the data dispersion is nearly equal along both axes; (2) plot the data points; (3) rotate the axes so as to exclude a linear trend; (4) approximate the data by a smooth curve. This procedure is efficient for monotonous, slightly curved regression lines (and may fail in more general cases). It can, however, produce major errors if data for one of the two magnitudes are truncated. A typical case is the  $m_b$  ( $M_s$ ) curve, which appears to be steeper than in reality at low  $M_s$  due to actual truncation of  $M_s$  data at about  $M_s = 5$ . We found no remedy for such cases and simply stopped our procedure when truncation seemed to be substantial.

Note that even in the case of linear regression, the orthogonal regression approach is not in fact theoretically founded, and presents only some reasonable recipe. In theory (e.g., YORK, 1966), deviations of data points from a "true"  $y(x)$  line are related to "inner" errors in  $x$  and  $y$ , and not to real deviations of sufficiently accurate values from the linear relation as in our case.

Surface wave magnitude  $M_s$  (either by vertical or by horizontal instrument) will be denoted as  $M_s^{\text{GR}}$  in the case employing the Gutenberg formula,  $M_s^{\text{US}}$  in the case of the “Prague” formula with  $T = 17\text{--}23$  s and corresponding amplitude (NEIC practice), and  $M_s^{\text{OB}}$  in the case of the “Prague” formula with maximum amplitude and corresponding period (ESSN practice, OB for Obninsk). Bodywave magnitude values determined by medium/long-period instruments are denoted as  $m_B$  as in PB. For magnitudes determined by short-period instruments we use  $m_b$  in the case of the Benioff instrument (NEIC), and  $m^{\text{SKM}}$  in the case of the SKM-3 instrument (ESSN);  $M_L$  is the original Richter magnitude based on the 1 s instrument, and  $M_{\text{JMA}}$  is the magnitude provided by the Japanese JMA service, based on the 5 s instrument.  $K^{\text{R60}}$  and  $K^{\text{F60}}$  are energy class scales after Rautian (RIZNICHENKO, 1960), used in many regions of the continental USSR, and after FEDOTOV (1972), used in Kamchatka. These are based on 0.7 s and 1.2 s instruments, correspondingly.

### *Long- and Medium-period Magnitudes*

The relationship  $M_s^{\text{GR}} (M_w)$  for large magnitudes was constructed based on  $M_s$  data of A for 1916–1980 and  $M_w$  of PB. Consequently it was reduced to  $M_s^{\text{US}} (M_w)$ , based on the relation  $M_s^{\text{US}} = M_s^{\text{GR}} + 0.18$  (see ABE, 1981), and the analysis was further carried out for  $M_s^{\text{US}}$  only. For small to moderate magnitudes we used catalogue  $M_s$  and  $M_o$  data of HAR and HEIC. The result is shown in Figure 1.

The relationship  $M_s^{\text{OB}} (M_w)$  was constructed in a similar way for small to moderate earthquakes. For large events data are scarce. The hypothesis  $M_s^{\text{OB}} \neq M_s^{\text{US}}$  was tested by using data from 1968–1983 and consequently rejected. In general, at  $M_s > 5$ , differences between  $M_s^{\text{US}}$  and  $M_s^{\text{OB}}$  are below 0.05, and we shall assume  $M_s^{\text{OB}} = M_s^{\text{US}}$ . Thus,  $M_s^{\text{OB}}$  branch is plotted in Figure 1 for  $M_s < 5$  only, where differences are significant. Up to  $M_s = 8.0$  our  $M_s^{\text{US}}$  vs  $M_o$  curve is naturally identical to that of EKSTRÖM and DZIEWONSKI (1988), including the useful relation

$$\log M_o = M_s^{\text{US}} + 19.24$$

for  $M_s^{\text{US}}$  range of 4–6.

The relationship  $m_B (M_w)$  for large magnitudes was established using again  $m_B$  from A and  $M_o$  from PB. For small to moderate magnitudes,  $m_B$  data from ESSN and  $M_o$  data from HAR were used. The result is seen in Figure 1; it can be tested by the substitution of the above-determined relation  $M_s^{\text{US}} (M_w)$ , with the correction of 0.18, into the formula (ABE and KANAMORI, 1980)

$$m_B = 0.65M_s^{\text{GR}} + 2.5;$$

giving similar results up to  $M_w = 8.7$ . However, the observed tendency to saturation at  $M_w \approx 9$  does not agree with the result of such substitution.

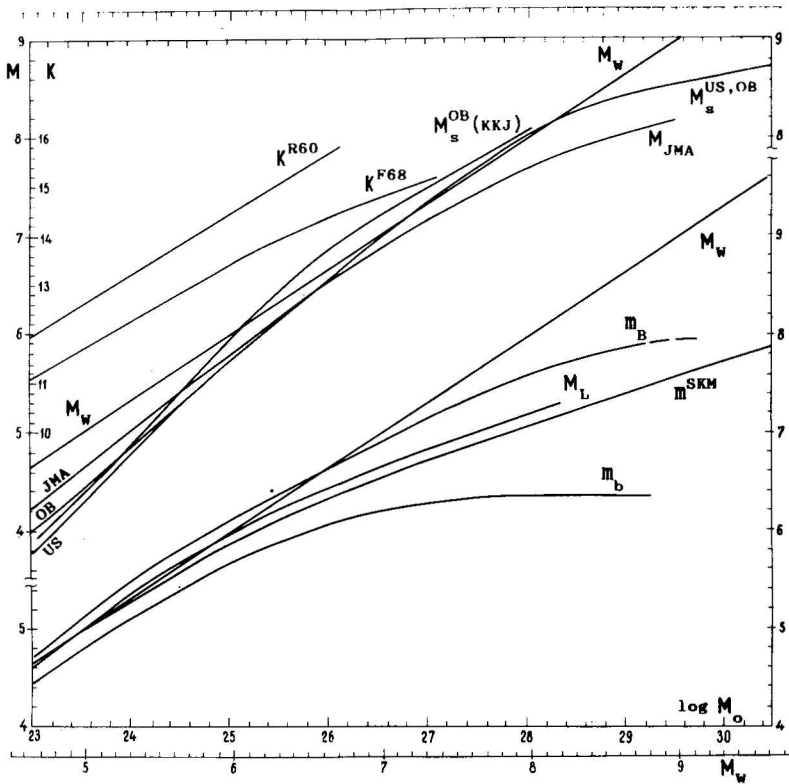


Figure 1

World average intermagnitude relationships presented with  $M_w$  (and  $M_o$ ) as an argument. Regional scales  $K$ ,  $M_L$  and  $M_{JMA}$  are also presented and the regional average  $M_s^{OB}$  curve for Kamchatka-Kuriles-Japan is shown too. To relate two magnitudes  $M_1$  and  $M_2$  one can proceed as  $M_1 \rightarrow M_w \rightarrow M_2$ . For notation see the text.

### Short-period Magnitudes

Short-period magnitude values depend on the precise value of the instrument period. A slight difference between Benioff and SKM-3 instruments leads to a minor difference between  $m_b$  and  $m^{SKM}$ , even at  $m_b < 5$  where the procedures for their determination more or less coincide. At  $m_b > 5.5$ , an additional requirement being used in the NEIC practice becomes essential, namely to select maximum amplitude only from several first peaks. As the true maximum normally arrives later, at large  $M_w$ ,  $m_b$  lacks clear meaning and saturates at the level of about 6.4. KOYAMA and ZHENG (1985) and also HOUSTON and KANAMORI (1986) redetermined  $m_b$  values for many large earthquakes using true maxima, and they denote the corrected  $m_b$  as  $m_b^*$  or  $\hat{m}_b$ . At first we carried out separate regression for  $m^{SKM}$

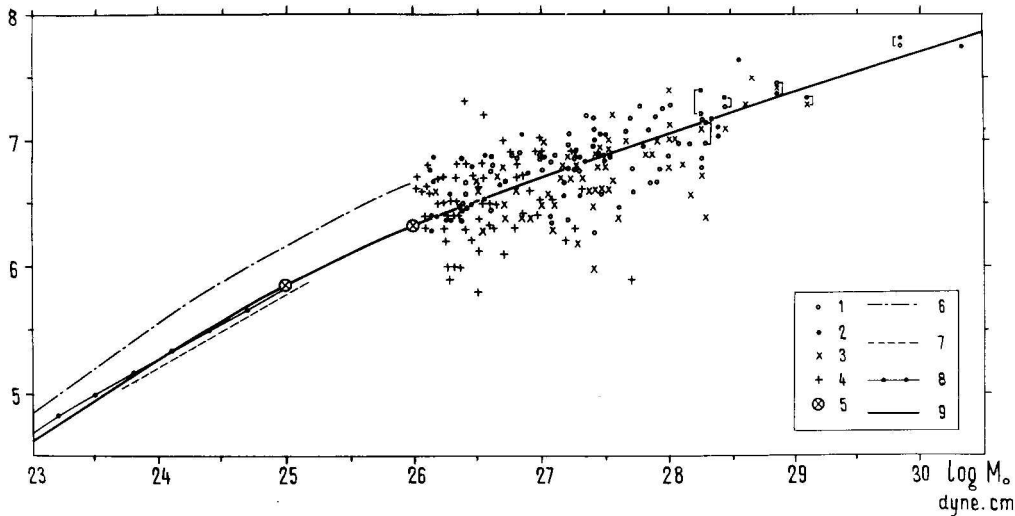


Figure 2

$m^{\text{SKM}}$  vs  $M_o$  relationship.  $m_b$ ,  $m_b^*$  and  $\hat{m}_b$  data are also given, with correction of +0.18. 1— $m_b^*$  after KOYAMA and ZHENG (1985); 2— $\hat{m}_b$  after HOUSTON and KANAMORI (1986) and ZHOU and KANAMORI (1987); 3— $m^{\text{SKM}}$ , mostly from PURCARU and BERCKHEMER (1981); 4— $m^{\text{SKM}}$  from ESSN catalogue and  $M_o$  from GIARDINI *et al.* (1985); 5—data centroids for  $m_b$  at given  $M_o$  from HAR; 6—“continental” trend of JOHNSON (1989); 7—“plate margin” trend of NUTTLI (1985); 8—trend for Kamchatka data partly using  $M_o$  from direct body waves; 9—accepted trend. Different estimates for the same event are indicated by a connection line.

and  $m_b^*$ , but discovered that regression curves practically coincide except for the constant difference

$$m_b^* = m^{\text{SKM}} - 0.18.$$

Thus, the plot in Figure 2 is given for  $m^{\text{SKM}}(M_o)$  or  $m_b^*(M_o) + 0.18$ . In the  $M_w = 6.6-9.5$  range it is well approximated by the straight line:

$$m^{\text{SKM}} = 0.525M_w + 2.86 = 0.35 \log M_o - 2.75.$$

Earlier values for the slope  $b$  of this line come from KOYAMA and SHIMADA (1985),  $b = 0.40$ , and from HOUSTON and KANAMORI (1986),  $b = 0.37$ .  $m^{\text{SKM}}$  and  $m_b$  curves are also plotted on Figure 1, the latter being based in part on BOORE (1986).

### Regional Scales

Regional scales  $M_L$ ,  $K^{\text{R60}}$ ,  $K^{\text{F68}}$ ,  $M_{\text{JMA}}$  are used in the regions of detailed seismological research. Their relationships versus  $M_w$  were mainly compiled. Note that the  $K$  scale, in its widely used versions, is a magnitude-type scale based on peak

amplitudes, and its relation to seismic energy is indirect (of regression type). In Figure 1 we present the  $M_L (M_w)$  curve based mainly on HANKS and BOORE (1984), the  $M_{JMA} (M_w)$  curve based on  $M_{JMA} (M_w)$  and  $M_{JMA} (M_s)$  plots of UTSU (1982), and the  $K^{R60} (M_w)$  curve according to RAUTIAN *et al.* (1981). As for  $K^{F68}$  vs  $M_w$ , this relationship was built up by the author. To obtain  $M_o$  values for this derivation we used HAR data,  $M_s$  data converted to  $M_o$  according to the world average relation, and  $M_o$  values determined by E. M. GUSEVA from the local records of direct body waves (observed on steep rays only, with clear unipolar body wave pulse).  $K^{F68}$  values from the catalogue of the Kamchatka regional seismic net were used at  $K^{F68} \leq 13$ . At larger  $K^{F68}$  values, standard regional instruments are largely off scale, and  $K^{F68}$  values determined routinely were demonstrated by V. M. ZOBIN to be inaccurate. Thus, instead of  $K^{F68}$  data, at  $K^{F68} > 13$  we used indirectly estimated  $K^{F68}$  values based on coda waves. The technique of such estimation was earlier developed by LEMZIKOV and GUSEV (1989) and is based on the coda wave level of a short-period instrument. To estimate the coda wave class  $K_c$  value, the following empirical formula was used

$$K_c = 1.6 \log A_c(100 \text{ s}) + \text{const},$$

where  $A_c(100 \text{ s})$  is coda amplitude (reduced or measured) at 100 s lapse time. LEMZIKOV and GUSEV (1989) have shown that  $K_c = K^{F68}$ , with an accuracy of about 0.4.

### *Intermagnitude Relations for the Kamchatka Region*

The above-presented relations are world-averaged, but there are more or less pronounced regional deviations from them. In the following we look for regional relationships for Kamchatka; earlier results were obtained by FEDOTOV (1972) and VIKULIN (1983).

Relationships  $M_s^{OB} (M_w)$  and  $M_s^{US} (M_w)$  were obtained for Kurile-Kamchatka (from data sources cited above) and for Japan (same sources and UTSU, 1982).  $M_s^{US}$  deviations from the average world trend are up to 0.1, while for  $M_s^{OB}$  they reach 0.35. In both cases, differences between the two regions appear to be negligible. The relationship  $M_s^{OB} (M_w)$  is plotted in Figure 1, marked KKJ.

Relationships of  $m_B$ ,  $m^{SKM}$  and  $m_b$  vs  $M_w$  for Kamchatka either coincide with the world average ( $m^{SKM}$ ,  $m_b$ ) or differ by a constant: regional  $m_B (M_w)$  is 0.15 above the average curve. A linear relation is found for  $K^{F68}$  when  $m^{SKM} = 4.3-7$ ;

$$K^{F68} = 2.00m^{SKM} + 1.68.$$

Note that  $K^{F68}$  roughly equals  $2 \log(A/T)$ , so that  $0.5K$  is somewhat of a local magnitude, and the above relation indicates the proportionality of short-period

local and teleseismic amplitudes despite the substantially larger bandwidth of a local record.

### *On the Accuracy of the Proposed Relationships*

The problem of accuracy of the presented set is complicated: a worldwide data set cannot be treated probabilistically as a statistical ensemble, because of pronounced regional variations. For the regional data set of Kamchatka, the standard deviations from average nonlinear trends (vs  $M_w$ ) are:  $\sigma(M_s^{OB}) = 0.35$ ,  $\sigma(M_s^{US}) = 0.20$ ,  $\sigma(m_B) = 0.25$ ,  $\sigma(m_b) = \sigma(m^{SKM}) = 0.30$ ,  $\sigma(K^{F68}) = 0.65$ . For  $K^{F68}$  vs  $m^{SKM}$ ,  $\sigma = 0.55$ .

Typical parameters of worldwide data sets are  $\sigma = 0.25-0.30$  for  $M_s$  ( $M_w$ ), and the same,  $\sigma = 0.25-0.30$ , for  $m^{SKM}$  ( $M_w$ ); but the actual values are strongly sample-dependent. Comparing regional and worldwide estimates one can suspect that interregional contribution to the variance is low. The actual picture is considerably more complicated because the contribution of subduction zones (mainly Pacific) to any random worldwide data set is dominating, and other tectonic types simply cannot manifest themselves. The most well-known regional effect is interplate-intraplate difference of  $\approx 0.25$  for  $m^{SKM}$  or  $m_B$ ; it can be seen in Figure 2. We may mention that we found the variations of the same magnitude when we divided the Kamchatka sample into subregions; thus the problem repeats itself on the more local level. As for the  $b$  value of the  $m^{SKM}$  vs  $\log M_o$  slope, its accuracy can be described roughly by  $\sigma = 0.02$ .

### *Spectral and Amplitude Trends of Source Radiation in the Short-period Spectral Band and Asperity Statistics*

We believe that the accurately determined value of the slope  $b$  of  $m^{SKM}$  vs  $\log M_o$  linear trend bears valuable information regarding properties of the earthquake source. For illustrative purposes one can compare our empirical estimate  $b = 0.35$  with the theoretical value  $b = 0.20$  obtained by HANKS and MCGUIRE (1981) for the combination of  $\omega^{-2}$  spectral model and the Gaussian process model of a record. Though they strove for a trend of peak acceleration, their result holds true for teleseismic amplitudes as well, as the numbers of peaks are comparable. The difference between the empirical and the theoretical estimates is drastic and it follows that at least one of the two theoretic assumptions must be rejected.

A short-period teleseismic  $P$ -wave record, the peak of which determines  $m^{SKM}$ , can be considered as a segment of narrow-band random signal with its central frequency close to 0.7 Hz. To study the scaling of peak amplitudes one can base on rms amplitude which can be related to the Fourier spectrum of a record, and further



to the source spectrum  $\dot{M}(f)$ . Hence we are interested in the estimates of the empirical trend of  $\dot{M}(f)|_{f=0.7 \text{ Hz}} = S_{0.7}$ .

Let  $\beta$  be the exponent in the relationship

$$\dot{M}(f) \propto M_o^\beta.$$

Data on  $\beta$  are scarce. GUSEV (1983) estimated the trend of a short-period spectral level from the trend of short-period body wave magnitudes; the slope of his linear relationship for the  $\log M_o = 26\text{--}29$  range indicates  $\beta = 0.37$  at 1 Hz. This estimate cannot now be considered as reliable because the Gaussian process model was assumed during its derivation. KOYAMA and ZHENG (1985) combined peak amplitude  $A_{\text{peak}}$  and duration  $d$  to determine the  $S_{0.7}$  vs  $M_o$  trend; they obtained the slope value  $\beta_{\text{KZ}} = 0.50$  for  $\log M_o = 25\text{--}30$ . They also studied the spectral level vs  $m_b^*$  relationship and found it to be linear in the wide range  $m_b^* = 5.5\text{--}7.5$ :

$$\log S_{0.7} = k_\beta m_b^* + \text{const}$$

with  $k_\beta = 1.24$ . All their results are biased since they implicitly assumed the rms amplitude to be proportional to peak one (or peak factor to be magnitude-independent). This is not actually the case, hence the direct use of numerical values of their  $\beta$  and  $k_\beta$  values is impossible. Their observation of linearity of  $\log S_{0.7}$  vs  $m_b^*$  trends in a rather wide magnitude range is, however, of vital importance.

One can try, however, to propose some interpretation for these data. Denote the peak factor  $PF = A_{\text{peak}}/A_{\text{rms}}$ , where  $A_{\text{rms}}$  is the rms amplitude, and let

$$PF \propto M_o^p.$$

Then  $\beta_{\text{KZ}} = \beta + p$  ( $p$  is nonnegative, so 0.50 and 1.24 are upper bound for  $\beta$  and  $k_\beta$ ). Now note that  $S_{0.7} \propto A_{\text{rms}} d^{0.5}$ , and let  $d \propto M_o^q$ , then  $A_{\text{rms}} \propto M_o^{\beta - 0.5q}$ . Combining this with  $A_{\text{peak}} \propto M_o^b$ , obtain

$$p = b - \beta + 0.5q$$

So that  $\beta_{\text{KZ}} = b - 0.5q$ . At  $\beta_{\text{KZ}} = 0.50$  and  $b = 0.35$  this gives  $q = 0.30$ . This is only slightly below  $q = 1/3$  which is the value that could be expected, based on the similarity assumption; the difference may represent the effect of scattering (which leads to  $q \approx 0$  at small enough  $M_o$ ).

Spectral trend was properly estimated first by HOUSTON and KANAMORI (1986) who found  $\beta = 0.45$  for 0.55 Hz based on a limited data set. ZHOU and KANAMORI (1987) revealed large data dispersion reflecting a tectonic setting of earthquakes and found  $\beta = 0.52$  for 1 Hz and  $\beta = 0.41$  for 0.5 Hz for subduction zone event subgroup of their data. Regrouping their data differently, one can obtain other figures; for the subgroup of plate margin earthquakes, for instance, we found that  $\beta$  nears 0.40 for both 1 and 0.5 Hz. Such freedom of interpretation seems to be produced by the mentioned data dispersion as well as by a relatively narrow magnitude range.



HARTZELL and HEATON (1985) studied Pasadena spectral levels and found the nonlinear trend of spectral level for the short to medium period range. For 0.7 Hz their spectral trend can be roughly described by two linear segments: one with  $\beta = 0.39$  for  $M_w = 7-8.5$  (53 events), and another with  $\beta = 0-0.1$  for  $M_w > 8.5$  (5 events). HARTZELL and HEATON (1988) try to confirm this saturation of the spectral level by the observation of a similar saturation for amplitudes; this trend is virtually equivalent to the  $m_B$  vs  $M_o$  relationship. The observed nonlinearity is equivalent to that seen in Figure 1 for  $m_B$ . For the frequency band of around 0.7 Hz, however, the analogue of  $m_b^*$  vs  $\log M_o$  relation given by HARTZELL and HEATON (1988) demonstrates no visible saturation. Data of KOYAMA and ZHENG mentioned above also show no saturation. Thus, we shall not base our estimates on HARTZELL and HEATON's (1985) data for  $M_w = 8.75-9.5$  range.

In general, the accuracy of the described  $\beta$  estimates is low. Since we believe that our estimate  $b = 0.35$  is rather accurate, we can try to determine  $\beta$  more accurately as  $k_\beta b$  if we manage to determine  $k_\beta$  accurately. One can assume *a priori* that the  $S_{0.7}$  vs  $\log M_o$  relationship will show a larger dispersion than  $m_b^*$  vs  $S_{0.7}$ , since spectral shape anomalies (including those related to tectonic setting) will be at least partially excluded in the latter case. The data generally confirm this idea: for  $\log M_1$  value of KOYAMA and ZHENG (1985) ( $M_1 \approx S_{0.7}$ ),  $\sigma(\log M_1) \approx 0.25$  on their  $M_1$  vs  $M_o$  plot and  $\sigma(\log M_1) \approx 0.20$  on their  $M_1$  vs  $m_b^*$  plot. Hence, certain variance reduction can truly be achieved. Thus, an indirect estimate of  $\beta$  can be more accurate.

Therefore, we compiled relevant data to estimate  $k_\beta$ . Data on  $S_{0.7}$  vs  $\hat{m}_b$  ( $\hat{m}_b \equiv m_b^*$ ) from HOUSTON and KANAMORI (1986) and ZHUO and KANAMORI (1987) are plotted in Figure 3. When no  $m_b^*$  was present we used  $m^{\text{SKM}} - 0.18$  instead. We make mention that plate interior earthquakes, with large positive deviations of their  $S_{0.7}$  values from the regression line for subduction zone or "plate margin" groups show no anomaly on  $S_{0.7}$  vs  $\hat{m}_b$  plot. The absolute level of  $S_{0.7}$  depends on particular assumptions made during inversion for source spectrum, mainly on  $t^*$ . HOUSTON and KANAMORI (1986) and ZHUO and KANAMORI (1987) used  $t^* = 0.7$  while HARTZELL and HEATON (1985) used  $t^* = 1.0$ . We plotted  $S_{0.7}$  values of the latter authors, averaged for several  $M_w$  intervals, both in original form and reduced to  $t^* = 0.7$  according to HOUSTON and KANAMORI (1986). We could not directly display the important data set of KOYAMA and ZHENG on the same plot because of calibration problems (they used roughly  $t^* = 1.4$ ) as well because of the aforementioned bias in their estimates. The role of these data, however, is important because of their large magnitude range. We plotted a segment of a straight line through the modified HARTZELL and HEATON's average point for  $m_b^* = 6.5$  ( $M_w = 7.25$ ). The length of this segment represents the data range of KOYAMA and ZHENG, and its slope  $k_\beta = 1.12$  equals the slope based on HARTZELL and HEATON's (1985)  $\beta = 0.39$  cited above. This  $\beta$  value will be accepted as our final estimate.

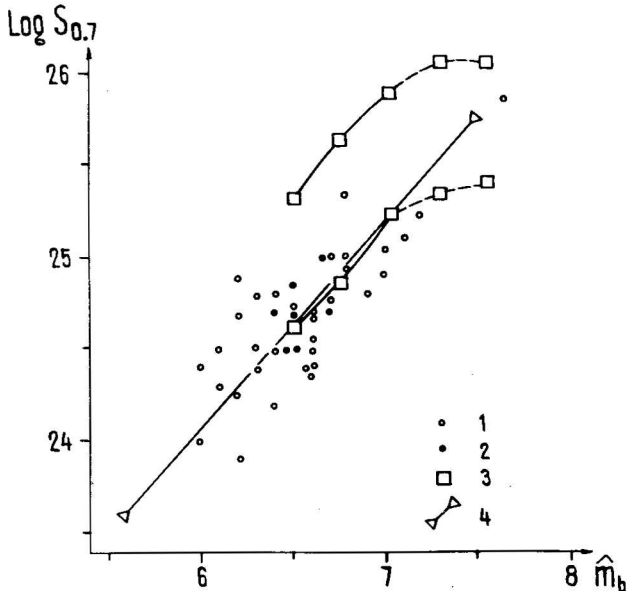


Figure 3

Source spectral level  $S_{0.7}$  vs  $m_b$  relationship. ( $S_{0.7} \equiv \dot{M}_o(f)|_{f=0.7 \text{ Hz}}$ ). 1—data of HOUSTON and KANAMORI (1986), ZHOU and KANAMORI (1987) and HWANG and KANAMORI (1989) for plate margin earthquakes; 2—same, plate interior; 3—data centroids of  $S_{0.7}$  after HARTZELL and HEATON (1988); original (the upper line) and modified by HOUSTON and KANAMORI (1986) (the lower line); 4—results of KOYAMA and ZHENG (1985), presented as a straight segment (see the text for details).

Now we can finally pass to source properties. Determining the peak factor value by the relation  $p = b - \beta + 0.5q$ , we obtain  $p = 0.13$  for the theoretical  $q = 1/3$  (or  $p = 0.11$  for the empirical  $q = 0.30$  based on KOYAMA and ZHENG (1985) data). Both values are fully incompatible with a Gaussian process record model which gives  $p \approx 0.03\text{--}0.04$  (HANKS and MCGUIRE, 1981), and this model can be completely rejected. GUSEV (1989) assumed the power-law statistics for amplitudes of individual acceleration pulses that combine into an accelerogram, and estimated the exponent of the law to be  $\alpha \approx 2$ . Similarly, we assume that the teleseismic short-period record is also produced by pulses with power-law statistics. For the peak amplitude  $A_p$ , it gives

$$A_p \propto N^{1/\alpha} \propto S^{1/\alpha} \propto M_o^{2/3\alpha},$$

where  $N$  is the number of pulses. This holds, if two assumptions are true:  $N$  is proportional to the source area  $S$ , and  $S$  value is proportional to  $M_o^{2/3}$ . Comparing the theoretical  $b = 2/3\alpha$  with the empirical  $b = 0.35$ , it results in  $\alpha = 1.9$ . From GUSEV (1989), pulse amplitude statistics of band-filtered displacement signal can represent statistics of seismic force  $F_o$  values of asperities ( $F_o = \int_{S_a} \Delta \sigma dS$ , where  $\Delta \sigma$

is the local stress drop and  $S_a$  the asperity area). For an asperity population with nearly constant  $S_a$  this indicates the power law distribution of average (over  $S_a$ )  $\Delta\sigma$  value, and  $\alpha \approx 2$  for this law. This conclusion coincides with the analogous result of GUSEV (1989) based on near-field data.

Another interesting point is the probable difference between empirical  $\beta$  estimate (0.39) and  $\beta = 1/3$  expected from the  $\omega^{-2}$  model. One cannot be sure at present that the difference is significant, because no accurate error estimate can be ascribed to our  $\beta$  value. Additional evidence in support of the reality of this difference can be found in regional scaling laws. PAPAGEORGIOU and AKI'S (1985) spectral scaling, based on the Western USA data, indicates  $\beta = 0.40$ . SUGITO and KAMEDA (1985) analyzed the Japanese data; their results indicate  $S_{0.7} \propto 10^{0.6M_{JMA}}$  for the magnitude range  $M_{JMA} = 5-8$ ; converting  $M_{JMA}$  to  $M_o$  by Figure 1, obtain  $\beta = 0.45$ . Thus we can consider the hypothesis  $\beta > 1/3$  as probable. Assuming  $\beta > 1/3$  one can try to explain the difference in frames of the multiasperity fault model of GUSEV (1989) if one assumes that the average asperity size  $2R_a$  is slowly growing with  $M_o$ ; actually  $\beta = 0.39$  can mean  $2R_a \propto M_o^{0.06}$ .

### *Discussion and Conclusions*

Curves of Figure 1 show several features of magnitude scales that were widely discussed in recent years: the slope value of 1 for  $M_s^{US}$ ,  $m_B$ ,  $m^{SKM}$ ,  $m_b$  vs  $\log M_o$  at low magnitudes, and saturation of  $m_b$  scale. Some new features are also emphasized:

- (1) coincidence of  $m^{SKM}$  and  $m_b^*$  (modified  $m_b$ ) scales up to a constant shift;
- (2) lack of real saturation for all scales excepting  $m_b$  (total saturation of  $m_B$  remains under question); instead of saturation, an interval of slow increase is present at the upper end of various curves;
- (3) similar shape of curves for short period scales:  $m^{SKM}$ ,  $M_L$  and  $0.5K^{F68}$ ;
- (4) good agreement between  $M_s^{OB}$  and  $M_s^{US}$  (which both do not directly match either  $M_s^{GR}$  or  $M_{GR}$ ). Disagreement is observed below  $M_s = 5$  only, reflecting the contribution of data from small epicentral distances when inadequate visual period is used in the "Prague" formula during the  $M_s^{OB}$  determination.

To simplify applications, a tabulated version of our results is added as Table 1.

In order to compare amplitude and spectral trends for short periods we need an estimate of the spectral trend and compiled spectral data from different sources. We believe that the proposed value  $\beta = 0.39$  is a reasonable starting point to deduce some conclusions regarding earthquake source. We found that the widely accepted assumption of the Gaussian noise model of short-period record is not supported by short-period teleseismic data. Some heavy-tailed distribution must be assumed to explain actual peak statistics.

Table 1

Magnitude vs seismic moment relationships: global average, for regional scales, and regional for Kamchatka-Kurile-Japan region (KKJ)

	log $M_0$ , dyne.cm							
	23	24	25	26	27	28	29	30
$M_s^{GR}$	3.58	4.58	5.54	6.34	7.12	7.82	8.23	8.45
$M_s^{US}$	3.76	4.76	5.72	6.52	7.30	8.00	8.41	8.63
$M_s^{OB}$	4.00	4.83	5.68	6.49	7.30	8.00	8.41	8.63
$m_B$	4.70	5.47	6.08	6.62	7.13	7.55	7.85	(7.98)
$m^{SKM}$	4.62	5.27	5.68	6.33	6.71	7.05	7.40	7.75
$m_b$	4.45	5.10	5.66	6.05	6.26	6.34	6.34	6.34
$M_L$	4.60	5.34	5.95	6.42	6.82	(7.16)		
$M_{JMA}$	4.22	4.99	5.77	6.49	7.12	7.64	8.04	(8.27)
$K^{F68}$	11.08	12.22	13.36	14.37	(15.11)	(15.80)		
$M_s^{US}$ (KKJ)	3.73	4.68	5.65	6.47	7.25	(7.99)		
$M_s^{OB}$ (KKJ)	3.84	4.84	5.95	6.84	7.48	(8.04)		
$M_w$	4.63	5.30	5.97	6.63	7.30	7.97	8.63	9.30

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